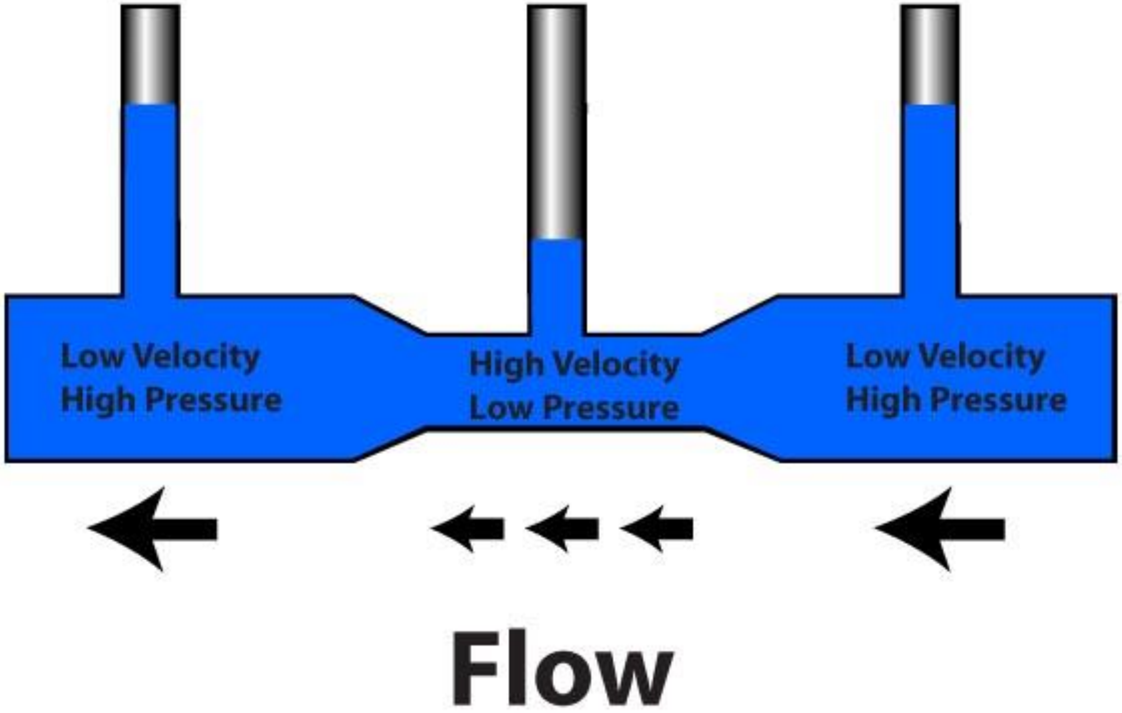
The Bernoulli Equation

Assumptions

- Steady flow (no change with time at a specified location) $\frac{d}{dt} = 0$
- Incompressible flow (Constant density) $\rho = const$
- Frictionless flow (Viscous effects are negligible compared to inertial, gravitational and pressure effects.) $\mu = 0$
- Irrotational flow (Applicable to inviscid regions of flow) $rot\vec{v} = o$

Bernoulli's Equation



- Bernoulli’s Equation
  - Kinetic Energy-velocity head
  - Pressure energy-pressure head
  - Potential Energy
- EGL/HGL graphs
  - Energy grade line
  - Hydraulic grade line

The Bernoulli Equation

By assuming that fluid motion is governed only by pressure and gravity forces, applying Newton's second law, F = ma, leads us to the Bernoulli Equation.

$$P/\gamma + V^2/2g + z = \text{constant along a streamline}$$

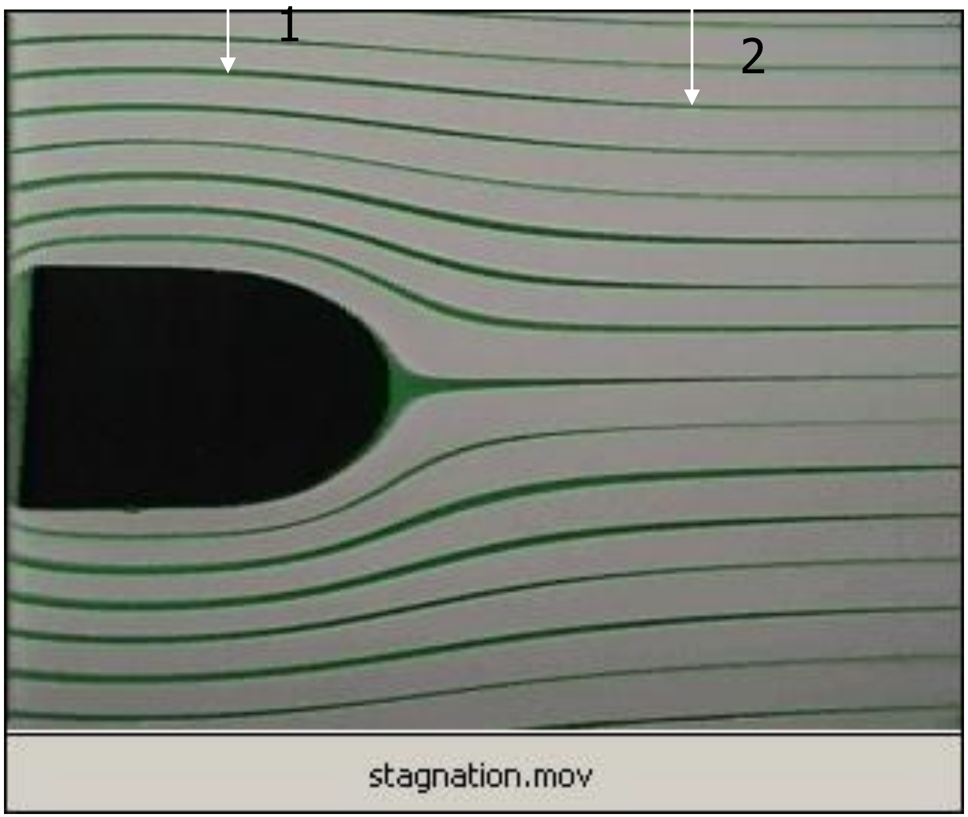
(P=pressure γ =specific weight V=velocity g=gravity z=elevation)

A streamline is the path of one particle of water. Therefore, at any two points along a streamline, the Bernoulli equation can be applied and, using a set of engineering assumptions, unknown flows and pressures can easily be solved for.

The Bernoulli Equation

At any two points on a streamline:

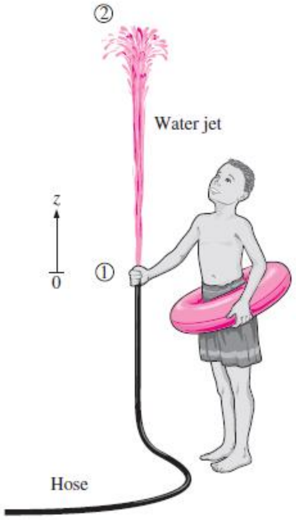
$$P_1/\gamma + V_1^2/2g + z_1 = P_2/\gamma + V_2^2/2g + z_2$$

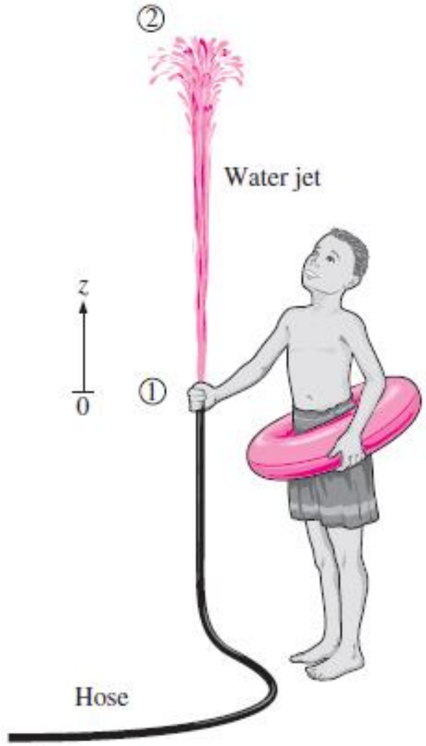


EXAMPLE 1 Spraying Water into the Air

Water is flowing from a hose attached to a water main at 400 kPa gage (Fig. below). A child places his thumb to cover most of the hose outlet, causing a thin jet of high-speed water to emerge. If the hose is held upward, what is the maximum height that the jet could achieve?

This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is relatively low ($V_1 = 0$) and we take the hose outlet as the reference level ($z_1 = 0$). At the top of the water trajectory $V_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli equation simplifies to





$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g}\cancelto{0}{} + \cancelto{0}{z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}\cancelto{0}{} + z_2 \quad \rightarrow \quad \frac{P_1}{\rho g} = \frac{P_{atm}}{\rho g} + z_2$$

Solving for z_2 and substituting,

$$z_2 = \frac{P_1 - P_{atm}}{\rho g} = \frac{P_{1,\,gage}}{\rho g} = \frac{400\ \text{kPa}}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1000\ \text{N/m}^2}{1\ \text{kPa}}\right)\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)$$

$$= 40.8\ \text{m}$$

Bernoulli Assumptions

There are three main variables in the Bernoulli Equation
Pressure – Velocity – Elevation

To simplify problems, assumptions are often made to eliminate one or more variables

Key Assumption # 1

Velocity = 0

Imagine a swimming pool with a small 1 cm hole on the floor of the pool. If you apply the Bernoulli equation at the surface, and at the hole, we assume that the volume exiting through the hole is trivial compared to the total volume of the pool, and therefore the Velocity of a water particle at the surface can be assumed to be zero

Key Assumption # 2

Pressure = 0

Whenever the only pressure acting on a point is the standard atmospheric pressure, then the pressure at that point can be assumed to be zero because every point in the system is subject to that same pressure. Therefore, for any free surface or free jet, pressure at that point can be assumed to be zero.

Key Assumption # 3

The Continuity Equation

In cases where one or both of the previous assumptions do not apply, then we might need to use the continuity equation to solve the problem

$$A_1V_1=A_2V_2$$

Which satisfies that inflow and outflow are equal at any section

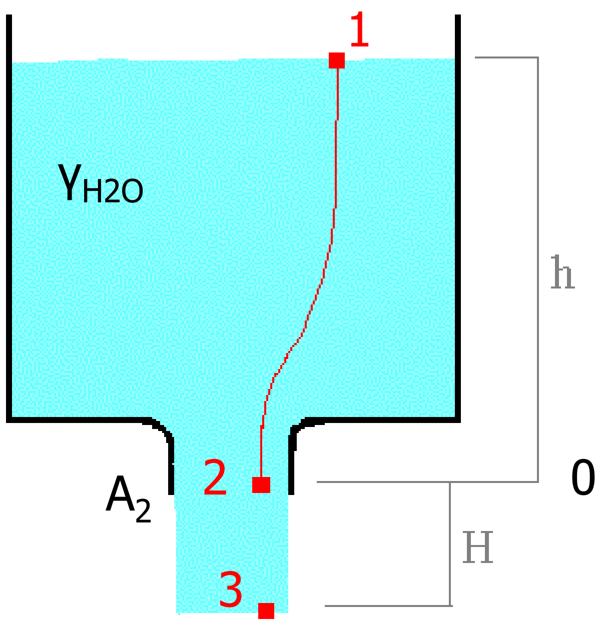
What is the Flow Rate at point 2? What is the velocity at point 3?

Givens and Assumptions:

Because the tank is so large, we assume $V_1 = 0$ ($Vol_{out} <<< Vol_{tank}$)

The tank is open at both ends, thus $P_1 = P_2 = P_3 = atm \rightarrow$

P_1 and P_2 and $P_3 = 0$



Part 1:

Apply Bernoulli's eqn between points 1 and 2

$$P_1/\gamma_{H2O} + V_1^2/2g + h = P_2/\gamma_{H20} + V_2^2/2g + 0$$

simplifies to

$$h = V_2^2/2g \quad \rightarrow \quad \text{solving for } V$$

$$V = \sqrt{(2gh)}$$

$Q = VA$ or $\mathbf{Q = A_2\sqrt{(2gh)}}$

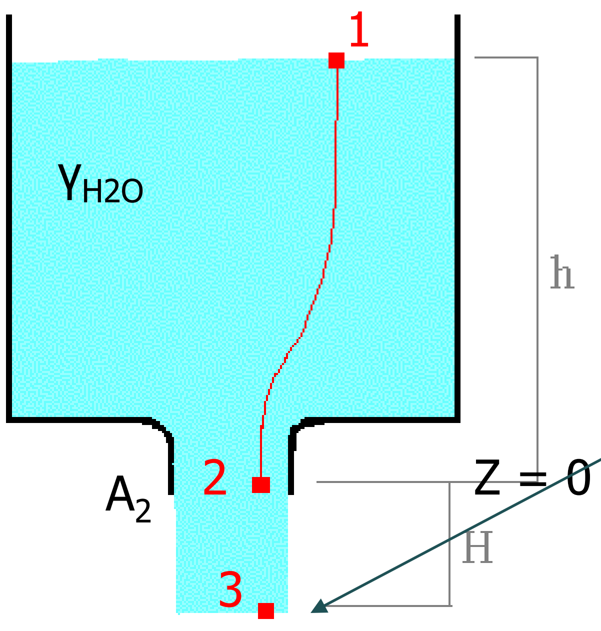
Part 2: Find V_3?

Apply Bernoulli's eq from pt 1 to pt 3

$$P_1/\gamma_{H20} + V_1^2/2g + h = P_3/\gamma_{H20} + V_3^2/2g - H$$

Simplify to → $h + H = V_3^2/2g$

Solving for V → $\mathbf{V_3 = \sqrt{(2g (h + H))}}$



Looking at the Bernoulli equation again:

$P/\gamma + V^2/2g + z =$ constant on a streamline

This constant is called the total head (energy), H

Because energy is assumed to be conserved, at any point along the streamline, the total head is always constant

Each term in the Bernoulli equation is a type of head.

P/γ = Pressure Head

Z = elevation head

Piezometric head

$V^2/2g$ = Velocity Head

These three heads summed equals H = total energy

Next we will look at this graphically…

Energy Grade Line (EGL)

Graphical representation of the total energy of flow of a mass of fluid at each point along a pipe.

EGL=HGL+Velocity Head

EGL=Potential+Pressure+Kinetic Energies

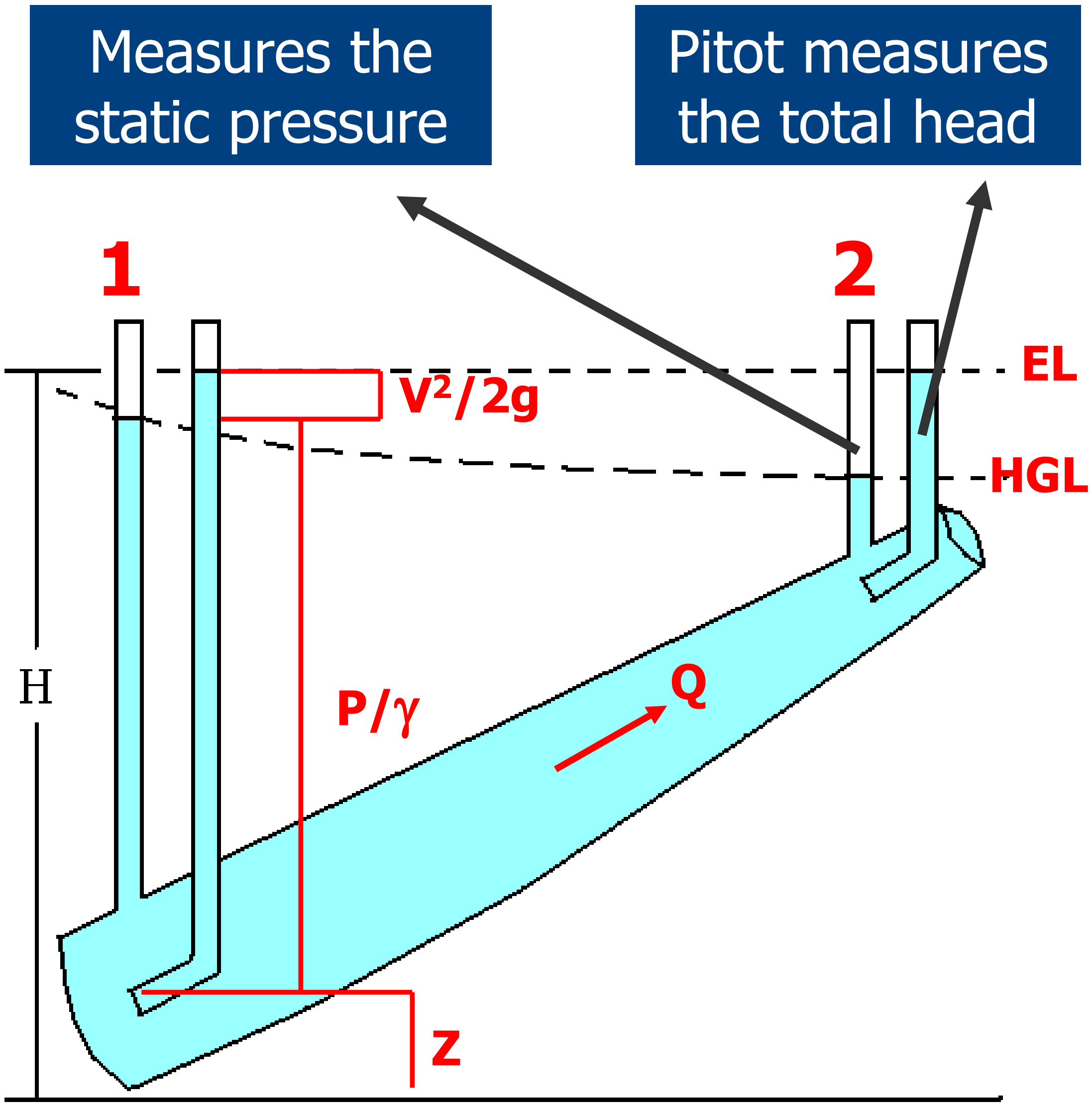
Hydraulic Grade Line (HGL)

Graphical representation of the elevation to which water will rise in a manometer attached to a pipe. It lies below the EGL by a distance equal to the velocity head.

EGL/HGL are parallel if the pipe has a uniform cross-section (velocity stays the same if Q & A stay the same).

HGL=Potential+Pressure Energies

The Energy Line and the Hydraulic Grade Line



1: Static Pressure Tap

Measures the sum of the elevation head and the pressure Head.

2: Pitot Tube

Measures the Total Head

EL : Energy Line

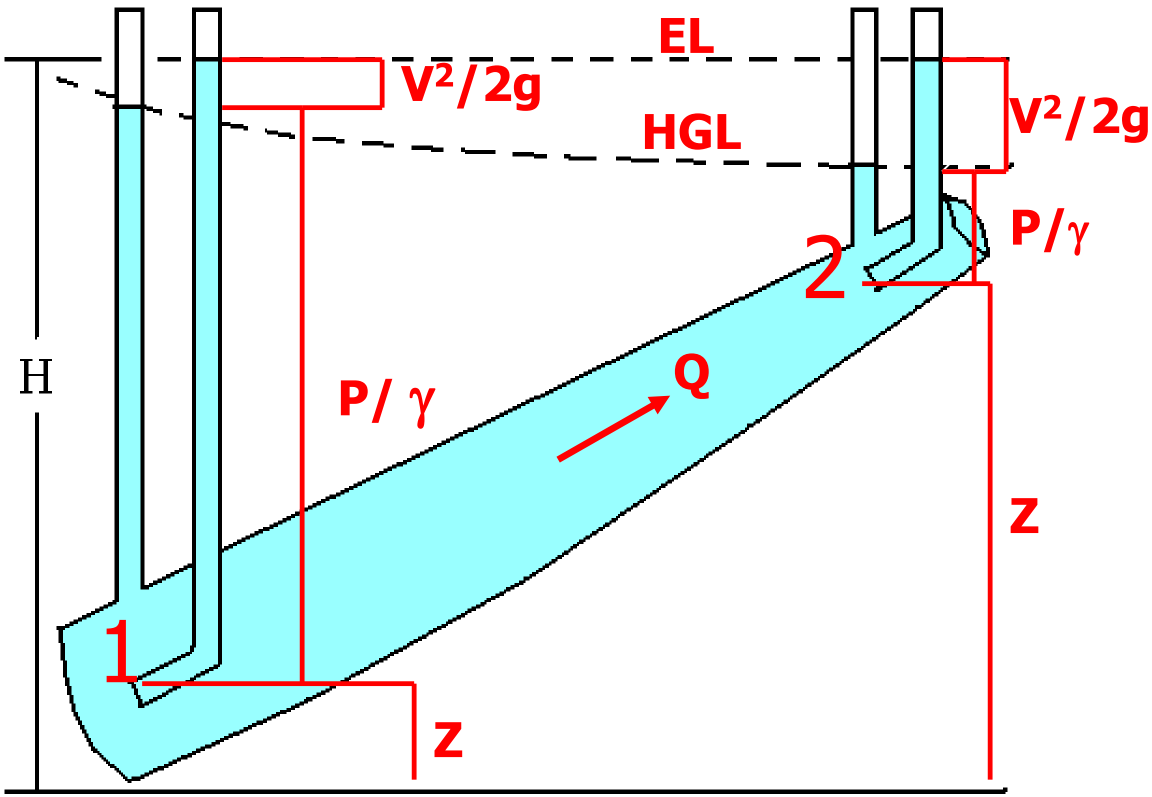
Total Head along a system

HGL : Hydraulic Grade line

Sum of the elevation and the pressure heads along a system

The Energy Line and the Hydraulic Grade Line

Understanding the graphical approach of Energy Line and the Hydraulic Grade line is key to understanding what forces are supplying the energy that water holds.



Point 1:

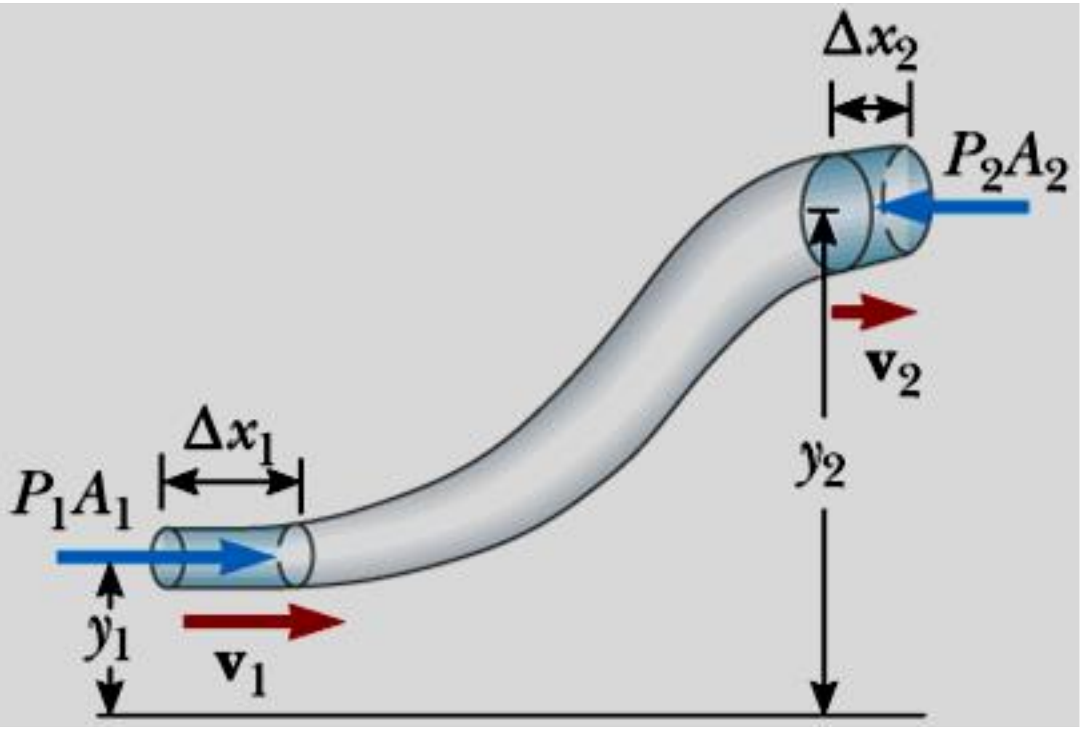
Majority of energy stored in the water is in the Pressure Head

Point 2:

Majority of energy stored in the water is in the elevation head

If the tube was symmetrical, then the velocity would be constant, and the HGL would be level

Bernoulli's equation



$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

$$P_1 + \frac{1}{2}\rho {v_1}^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho {v_2}^2 + \rho g y_2$$